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Trapped Particles at a  
Magnetic Discontinuity

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Abstract

At a tangential discontinuity between two constant magnetic fields a layer of trapped particles can exist ; this work examines the conditions under which the current carried by such particles tends to maintain the discontinuity (neglecting finite gyroradius effects). Three cases are examined. If the discontinuity separates aligned vacuum fields, the only requirement is that they be antiparallel. With arbitrary relative orientations, the field must have equal intensities on both sides. Finally, with a guiding center plasma on both sides, the condition reduces to the continuity of  $B^2(1 + \beta)$  , a relation which is also derivable from hydromagnetic theory. Arguments are presented for the occurrence of such trapped modes in the magnetopause and for the non-existence of specular particle reflection there, <sup>which was</sup> postulated by earlier theories.

According to classical electrodynamics a tangential discontinuity between vacuum magnetic fields  $\underline{B}_1$  and  $\underline{B}_2$  must contain a sheet current with a surface current density

$$\underline{J} = \hat{\underline{x}} \times (\underline{B}_1 - \underline{B}_2) / \mu_0 \quad (1)$$

where  $\hat{\underline{x}}$  is orthogonal to the discontinuity in the direction leading into the medium with field  $\underline{B}_1$ .

In general a tangential discontinuity can also support a trapped layer of charged particles. The purpose of this note is to investigate (under certain simplifying assumptions) the conditions which are required for this layer to be able to provide the sheet current of equation (1). The following cases may be distinguished:

(1)  $\underline{B}_1$  AND  $\underline{B}_2$  ALIGNED

Assume that  $\underline{B}_1 - \underline{B}_2$  is in the same direction as  $\underline{B}_1$ : two cases then may exist, depending on whether  $\underline{B}_1$  and  $\underline{B}_2$  are antiparallel or parallel. As shown in figure (1), in the first case all charged particles move in such a direction that their motion contributes a current parallel to  $\underline{J}$ , while in the second case they all move in the opposite direction. Thus only with antiparallel fields can the particle layer maintain the discontinuity.

It should be pointed out here that even though a sharp field discontinuity is assumed, the current flow contributed by trapped particles

of any energy will be spread out over a thickness equal to the sum of their gyroradii in the two media. This spreading out will be ignored in the present work, but it should be taken into account if the detailed structure of the discontinuity is required.

It may also be noted here that the trapped mode shown in figure (1-a) was used by Cowley [1972] in a model of the plasma sheet existing in the earth's magnetospheric tail. Since the field in the plasma sheet appears to vary on a scale considerably larger than the gyration radii of particles found there, it is not clear how well this application is justified. The present work will only deal with applications to the magnetopause, where the change in most cases is sufficiently abrupt, at least for protons.

(2)  $\underline{B}_1$  AND  $\underline{B}_2$  NOT ALIGNED

In this case it may be shown that the average motion of all trapped particles has the same direction and that this direction coincides with that of  $\underline{J}$  if and only if the field intensities are equal.

Let a given particle enter  $\underline{B}_1$  with initial velocity

$$\begin{aligned}\underline{v} &= v_1 \hat{\underline{x}} + v_2 \hat{\underline{B}}_1 + v_3 (\hat{\underline{x}} \times \hat{\underline{B}}_1) \\ &= \underline{v}_1 + v_2 \hat{\underline{B}}_1\end{aligned}\tag{2}$$

and suppose that in the plane orthogonal to  $\underline{B}_1$  the particle describes an arc of a length of either  $2\lambda$  or  $2\pi - 2\lambda$ , where in any case  $\lambda$

is between 0 and  $\pi/2$  (figure 2). It is then easily seen that  $\lambda$  complements the angle between  $\hat{x}$  and  $\underline{v}_\perp$ , giving

$$\sin \lambda = (\underline{v}_\perp \cdot \hat{x}) / v_\perp = v_1 / v_\perp \quad (3)$$

If  $\omega_1 = e B_1 / mc$  is the angular velocity of gyration in  $B_1$ , the gyration radius is  $\rho_1 = v_\perp / \omega_1$  and the displacement of the particle in one excursion amounts to

$$2 \rho_1 \sin \lambda (\hat{x} \times \hat{B}_1) = (2 v_\perp / \omega_1) (\hat{x} \times \hat{B}_1) \quad (4)$$

The particle crosses the boundary into the other field (denoted  $B_2$ ) with  $v_1$  reversed and the other two velocity components intact. It is then turned towards the sheet and stays trapped in its vicinity, but it is also displaced along the sheet in some direction, which on the long-term average is the same as the direction of the particle's displacement after two steps, one on each side of the discontinuity.

The particle's displacement on the side with  $x < 0$  will be along  $-(\hat{x} \times \hat{B}_2)$ , since now  $-\hat{x}$  is the unit vector orthogonal to the boundary and pointing into the region of the given field. By the preceding equation, this displacement therefore is

$$(- 2 v_1 / \omega_2) (\hat{x} \times \hat{B}_2) \quad (5)$$

The total displacement in one "double step" along the sheet is therefore

$$2 v_1 \hat{x} \times \left[ (\hat{B}_1/\omega_1) - (\hat{B}_2/\omega_2) \right] \quad (6)$$

All particles are displaced in the same direction, and this will be aligned with  $\underline{J}$  if and only if  $B_1 = B_2$ .

### (3) DISCONTINUITY AT THE BOUNDARY OF A PLASMA

Let each of the bounding media be filled with a guiding center plasma and let the ratio of particle kinetic energy density to magnetic energy density in each be denoted by  $\beta_i$ , where  $i$  is the index appropriate to the medium. Because of the diamagnetic properties of the plasma, equation (1) must be modified to

$$\underline{J} = \hat{x} \times (\underline{H}_1 - \underline{H}_2) \quad (7)$$

where

$$\mu_0 \underline{H}_1 = \underline{B}_1 - \underline{M}_1 \quad (8)$$

$\underline{M}_1$  being the volume magnetization of the appropriate medium. In a guiding center plasma immersed in a constant field [e.g. Longmire, 1963, sect. 2-3 ; Northrop, 1963, sect. 4-A]

$$\underline{M}_1 = - \underline{B}_1 \beta_1 \quad (9)$$

and hence

$$\underline{J} = \hat{x} \times \left[ \underline{B}_1(1 + \beta_1) - \underline{B}_2(1 + \beta_2) \right] \quad (10)$$

If the boundary current between the media is in the direction given by (6) , the cross product between the vectors inside the square brackets in equations (6) and (10) must vanish. This gives, after a few steps

$$B_1^2(1 + \beta_1) = B_2^2(1 + \beta_2) \quad (11)$$

This is not an unexpected result — it may also be derived from hydromagnetic considerations (balance of the sum of particle pressure and magnetic pressure) and has in fact been tested (with only partial success) against magnetopause data by Ogilvie et al. [1971] . What is new here is the derivation from the point of view of particle motion (even though considerable simplification is involved) and the conclusion that any relative orientation of  $\underline{B}_1$  and  $\underline{B}_2$  can be maintained by a trapped particle layer. In a vacuum, of course, both  $\beta_1$  and  $\beta_2$  vanish and the result reduces to the one obtained in the preceding section.

Suppose that  $\underline{B}_1$  and  $\underline{B}_2$  are aligned: then equation (10) shows that  $\underline{J}$  might have a direction opposite to that of  $\underline{B}_1 - \underline{B}_2$  . If this is the case, the conclusions derived in case (1) are reversed and the fields may be parallel as in figure (1-b), leading to an "overshooting" mode of motion, while the motion shown in figure (1-a) does not take place. Such modes may well be responsible for tangential discontinuities observed in the solar wind in which the field's direction remained unchanged [ Burlaga, 1971, fig. 20 ]

## APPLICATION TO THE MAGNETOPAUSE

Existing theories of the magnetopause [Willis, 1971 and references cited there] generally assume an electrically neutral beam of protons and electrons arriving at its surface in a parallel stream, which suffers specular reflection at the discontinuity of the field. Because protons have a much larger gyroradius than electrons, they are expected to penetrate more deeply across the boundary and to create, by means of their positive charge, a polarization field which greatly modifies the motion of particles inside the boundary layer itself. It has also been suggested that such electric fields might be neutralized by the high conductivity of the ionosphere, to which the boundary is linked along magnetic field lines. In fact, an experimental attempt to detect an electric field of this kind [Fairfield et al., 1972] has given negative results.

However, all such theories fail to treat the motion of charged particles in a realistic manner, consistent with the existence of a magnetic field on both sides of the boundary. Consider the neutral beam impinging on the magnetopause: the reason why its particles move in straight lines and do not spiral in the ambient field (in this particular model of the motion) is that one deals here with a high- $\beta$  plasma which dominates the magnetic field rather than being dominated by it. This "domination" is achieved by an induced electric field

$$\underline{E} = - \underline{v} \times \underline{B} \quad (12)$$



As long as all the particles have the velocity  $\underline{v}$ , the electric and magnetic forces on them balance each other, allowing them to move in straight lines.

Once a particle is reflected from a field discontinuity, however, this balance is destroyed, because the velocity vector is changed. In fact one finds that, instead of coming back along the directions of specular reflection, the particles will remain trapped in the vicinity of the discontinuity, moving in one of the modes discussed earlier.

The matter does not end here, however. Up to this point it has been assumed that  $\underline{E}$  of equation (12) extends all the way to the magnetopause. If this is the case, the electric field would accelerate or decelerate trapped particles as they move along the magnetopause and this, in turn, would serve to diminish  $\underline{E}$  near the boundary. However, once  $\underline{E}$  is reduced below the value given by (12) (with  $\underline{v}$  the velocity of arrival), incoming particles will no longer move in straight lines but will tend to spiral in the surrounding magnetic field. The motion of the particles when final equilibrium (if it exists) is achieved is not too clear : instead of an aligned beam hitting the boundary, one might well wind up near the boundary with an almost isotropic plasma and only a weak electric field.

Such a situation would resemble case (3) discussed in the preceding section. Equation (11) should then be satisfied, within the validity of the assumptions made here. As noted earlier, this relation was tested (for a few cases) by Ogilvie et al. [1971] but agreement was incomplete,

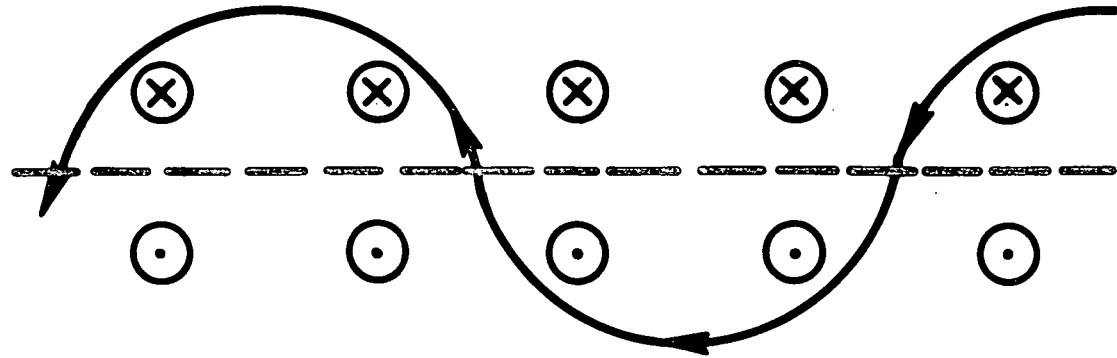
possibly because observations of the energy spectrum inside the boundary did not extend high enough to allow  $\beta$  to be accurately determined.

Also, if this view of the magnetopause is correct, there should not exist any component of  $\underline{B}$  orthogonal to the boundary. An attempt by Sonnerup and Cahill [1967, 1968] to detect such a component has yielded only a small value, which could be due to the low resolution of the observations.

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(a)



(b)

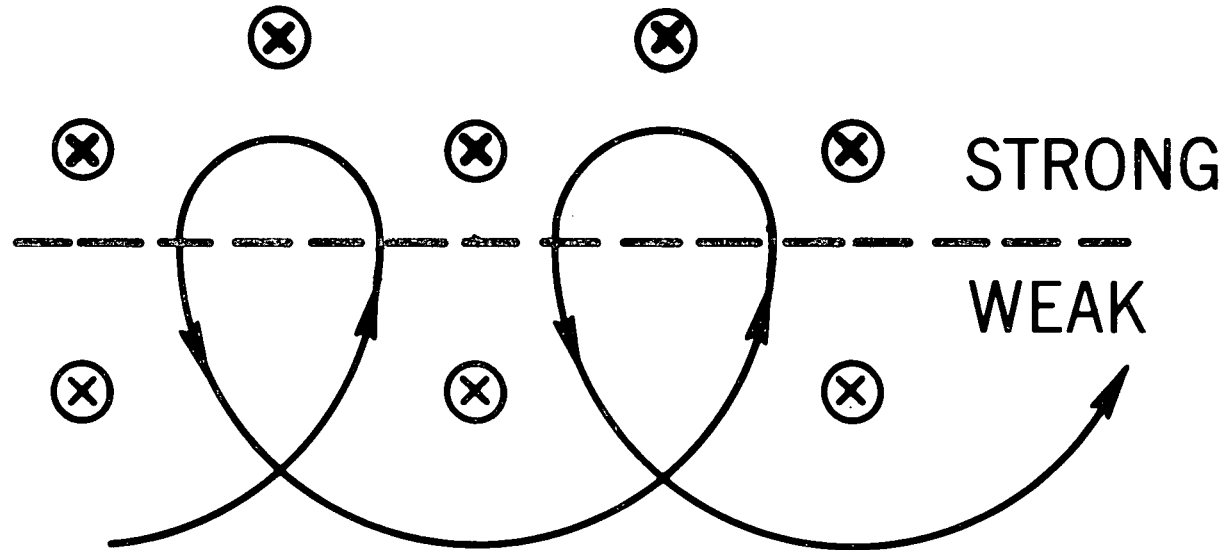


Figure 1

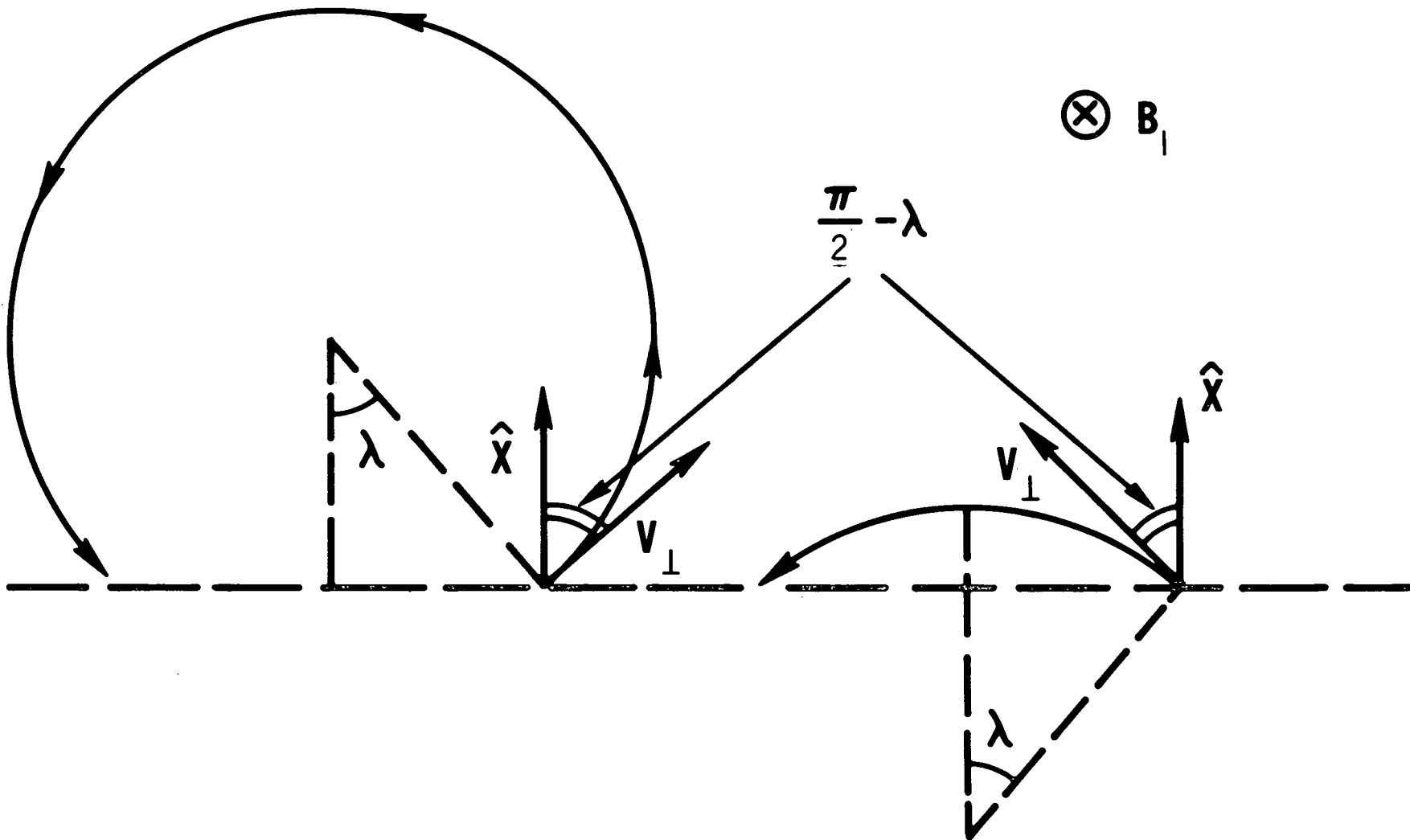


Figure 2

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